# OPTIONS EVALUATION - BLACK-SCHOLES MODEL VS. BINOMIAL OPTIONS PRICING MODEL

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## 1. Introduction

A particularly important issue that arises when it comes to options is fixing their value. The emergence of quantitative techniques that allow operators to follow the evolution of financial assets price has multiplied the transactions on futures markets.

Evaluation options theory has its roots in Bachelier's research (1900) who used Brownian motion to evaluate French options on government bonds. Only in the early '70s options valuation methods have begun to gain consistency by determining a formula for calculating the price of European options by Fischer Black and Myron Scholes.

Black and Scholes (1973) are the pioneers in pricing option theory. They started from the premise that if options are properly evaluated, there can be certainly no gain from the sale and purchase of options and underlying assets. Using this principle, they introduced a formula for determining the theoretical value of an option. This model is the starting point for most further research. So Broadie, Detemple, Ghysels and Torres (2000) determine the price of American options when the asset's support provides a stochastic dividend yield. They show that an American option is worth the sum of the European option premium and the premium for exercising the option before maturity. Chance, Kumar and Rich (2002) provided conditions under which the standard formula Black-Scholes-Merton is valid, even if the dividend is stochastic. To do this, they assumed that the present value of future dividends is observable and a forward contract can be obtained by trading these accumulated dividends. To obtain an explicit formula for determining the option price when the market is incomplete, Geske (1978) used the CAPM (Capital Asset Pricing Model) to balance the risk premium in the economy. From Black-Scholes-Merton model, Lioui A. (2006) obtained new formulas to evaluate options by considering the stochastic dividend yield. Some of the hypotheses Black-Scholes-Merton model are removed under the new approach.

Unlike Black and Scholes who used the principle of continuous valuation, Cox, Ross and Rubenstein designed the binomial model for calculating the price of an American option, based on the approximation of a continuous process with a discreet one. This model was presented in 1979 in *Option Pricing: A Simplified Approach.* Model summary consists in simulation of underlying asset price evolution by dividing the time to maturity in a certain number of short periods. Binomial method is useful and very popular for American call and put on a stock providing dividends. The basic principle of this model is that the underlying asset price can either increase or decrease in the next period.

## 2. Black-Scholes formula for determining the price of European options

Black-Scholes model for determining the price of a European option is widely used in practice because it requires knowledge of observable parameters: the underlying asset price, the strike price, the time to maturity of the option, the continously compounded risk-free rate and a parameter to be estimated independently, the underlying assets volatility. The model is based on a set of assumptions of which the most restrictive are: the underlying asset yield are normally distributed, volatility remains constant throughout the life of the option, there are no transaction costs and it can borrow money at the risk free rate.

Starting price evaluation model of European options by Black and Scholes developed in the '70s, it was two formulas for determining the price a European call option (c) and a European put option (p):

$$c = S_0 N(d_1) - X e^{-r(T-t)} N(d_2)$$

$$p = -S_0 N(-d_1) + X e^{-r(T-t)} N(-d_2),$$

where

$$d_1 = \frac{\ln(S_o / X) + (r + \sigma^2/2)(T-t)}{\sigma\sqrt{T-t}}$$

$$d_2 = \frac{\ln(S_o / X) + (r - \sigma^2/2)(T-t)}{\sigma\sqrt{T-t}} = d_1 - \sigma\sqrt{T-t}$$

The function N(x) is the cumulative probability distribution function for a standardized normal distribution. In other words, it is the probability that a variable with a standard normal distribution with average 0 and standard deviation 1. $S_0N(d1)$ is the present value of the asset if the option is exercised, and $Xe^{-r(T-t)}$ is the present value of the strike price if the option is exercised.

$S_0$ – the stock price at time 0;

X – the strike price;

σ – the underlying assets volatility;

r – the continously compounded risk-free rate;

T-t – the time to maturity of the option. T is the maturity, and t is the moment to maturity (at option issue t=0).

Previous formulas are used when the underlying asset does not generate earnings. The amount of dividends generated by the underlying asset of the option affects its price and, accordingly, the option price. Thus, the posting of dividends results in lower premium for call options, and to increase for the premium for put options. If the underlying asset generates earnings, formulas for determining the price of an option are:

$$c = S_0 e^{-q(T-t)} N(d_1) - X e^{-r(T-t)} N(d_2)$$

$$p = -S_0 e^{-q(T-t)} N(-d_1) + X e^{-r(T-t)} N(-d_2)$$

where

$$d_1 = \frac{\ln(S_o / X) + (r - q + \sigma^2/2)(T-t)}{\sigma\sqrt{T-t}}$$

$$d_2 = \frac{\ln(S_o / X) + (r - q - \sigma^2/2)(T-t)}{\sigma\sqrt{T-t}} = d_1 - \sigma\sqrt{T-t}$$

,

and q is the annual dividend yield if the underlying asset is a share or an index and the risk-free asset rate of foreign currency if the underlying asset is the exchange rate.

Existence of an analytical solution for the price of a European option allows analyzing how their prices respond to changes of variables and parameters is determined. It's the Black-Scholes model assumptions, the variables: the underlying price (S) and time to maturity (T-t) and the parameters: the underlying assets volatility (σ), the continously compounded risk-free rate (r) and the strike price (X).

Options price response to the changes of these variables are virtually the sensitivity coefficients of the premium and main elements for measuring the risk that these financial assets involve and are used to define practices cover such risks. In addition, the indicators facilitate the development of cash flows generated by derivative in the underlying asset trading, technique which can be useful if certain financial portfolio management strategies involve derivatives.

Given the importance of knowing the sensitivity indicators, we will continue this way of determining their values in relation to different variables and their use.

**Delta (Δ)** is the most famous on Greek letters and it measures the option price sensitivity to variations in the price of the underlying asset.

Delta is calculated as the first derivative of on option price to a change in the price of the underlying asset when the other parameters remain constant. Practically, delta is the number of units of the underlying asset we should hold for each option shorted in order to create a riskless hedge.

For a European option on a non-dividend-paying stock, it can be shown that:

$$\Delta_c = \frac{\partial c}{\partial S} = N(d_1)$$

$$\Delta_p = \frac{\partial p}{\partial S} = N(d_1) - 1$$

The delta of a call option is always positive, that is to say, a variation in the price of the underlying asset implies a variation, in the same direction, in the price of the call option. On the other hand, the value of a put option decreases if the price of the underlying asset increases, implying, therefore, a negative delta.

For European options on an asset paying a yield q, we have:

$$\Delta_c = \frac{\partial c}{\partial S} = e^{-q(T-t)} N(d_1)$$

$$\Delta_p = \frac{\partial p}{\partial S} = e^{-q(T-t)} [N(d_1) - 1]$$

**Gamma (Γ)** measures the delta sensitivity to changes in the underlying asset and it is represented mathematically as second derivative of option price to underlying price or first order derivative of the delta to S. Gamma is identical for both call and the put option and can be positive or negative. For a European option on a non-dividend-paying stock, it can be shown that:

$$\Gamma_c = \frac{\partial \Delta_c}{\partial S} = \frac{\partial^2 c}{\partial S^2} = \frac{N'(d_1)}{S\sigma\sqrt{T-t}}$$

$$\Gamma_p = \frac{\partial \Delta_p}{\partial S} = \frac{\partial^2 p}{\partial S^2} = \frac{N'(d_1)}{S\sigma\sqrt{T-t}}$$

where, $N'(x) = \frac{1}{\sqrt{2\Pi}} e^{-\frac{1}{2}x^2}$

For a European call or put option on an asset paying a continous dividend at rate q:

$$\Gamma_c = \Gamma_p = \frac{N'(d_1) * e^{-q(T-t)}}{S\sigma\sqrt{T-t}}$$

Geometric, gamma is the slope of the graph Δ = *f(S)* or convexity option price to underlying price changes, was used to measure risk coverage, so the risk position delta neutral.

Gamma presents values close to zero when the option is out of the money or in the money and the maximum value when the option is at the money, especially when the time to maturity is reduced.

**Theta (Θ)** measures the option price change over time, while the other parameters are constant. To determine the theta of a European option on a non-dividend-paying stock, there are using the following formulas**:**

$$\Theta_c = -\frac{\partial c}{\partial (T-t)} = -S\,\frac{N'(d_1)\sigma}{2\sqrt{T-t}} - rXe^{-r(T-t)}N(d_2)$$

$$\Theta_c = -\frac{\partial p}{\partial (T-t)} = -S\,\frac{N'(d_1)\sigma e^{-q(T-t)}}{2\sqrt{T-t}} - qSN(-d_1)e^{-q(T-t)} + rXe^{-r(T-t)}N(-d_2)$$

For a European call or put option on an asset paying a continous dividend at rate q:

$$\Theta_c = -\frac{\partial c}{\partial (T-t)} = -S\,\frac{N'(d_1)\sigma e^{-q(T-t)}}{2\sqrt{T-t}} + qSN(d_1)e^{-q(T-t)} - rXe^{-r(T-t)}N(d_2)$$

$$\Theta_c = -\frac{\partial p}{\partial (T-t)} = -S\,\frac{N'(d_1)\sigma e^{-q(T-t)}}{2\sqrt{T-t}} - qSN(-d_1)e^{-q(T-t)} + rXe^{-r(T-t)}N(-d_2)$$

Theta is in most cases a negative parameter, because as they approach maturity, option value tends to decrease. Exception to this observation are European put options on shares that have a very strong position in the money, or call options in the money on currencies that have a very high interest rate.

**Vega (K).** Black-Scholes formula has been demonstrated in the constant volatility underlying assumption for the time the option price is calculated. In practice, volatility is a variable parameter, which determines the option price changes. These changes are calculated using vega which represents the first derivative of an option value to volatility.

For a European option, a call or a put, having an underlying asset that generates no gain, coefficient is given by the following formula:

$$K_c = \frac{\partial c}{\partial \sigma} = S\sqrt{(T-t)}N^{'}(d_1)$$

$$K_p = \frac{\partial p}{\partial \sigma} = S\sqrt{(T-t)}N^{'}(d_1)$$

For a European call or put option on an asset providing a dividend yield at rate q:

$$K_c = \frac{\partial c}{\partial \sigma} = S\sqrt{(T-t)}N^{'}(d_1)e^{-q(T-t)}$$

$$K_p = \frac{\partial p}{\partial \sigma} = S\sqrt{(T-t)}N^{'}(d_1)e^{-q(T-t)}$$

Vega is always a positive coefficient and a higher value indicates a high sensitivity to option volatility changes. If vega has low volatility changes, it will have a little impact on option price. Strong out or in the money options have a low vega, and the one at the money high, especially when maturity is delayed.

**Rho (P)** measures the sensitivity of the option value of interest rate and it is calculated as the first derivative of option price to interest rate. For a European option, a call or a put, having an underlying asset that generates no gain, the coefficient is given by the following formula:

$$\mathrm{P}_c = \frac{\partial c}{\partial r} = X(T-t)e^{-r(T-t)}N(d_2)$$

$$\mathrm{P}_p = \frac{\partial p}{\partial r} = -X(T-t)e^{-r(T-t)}N(-d_2)$$

The same formula applies if the underlying European option generates dividend.

Rho is always positive for a call option, while for a put option the coefficient is negative.

**Example no. 1**: Consider a European call option and a European put option on a stock that generate dividend and it has the following characteristics: S=10 RON, E=11 RON, r=8.5%, σ=22%, T=3 months, q=2%.

Applying the previous formulas, we obtain:

Call premium=0.157455

Put premium=0.976046

**Determination of sensitivity coefficients of an option**

| | Call | Put |
|---|---|---|
| Delta | 0.252167 | -0.7428446 |
| Gamm | 0.2895241 | 0.2895241 |
| Vega | 0.01592383 | 0.01592383 |
| Theta | -0.002332 | -0.0003694 |
| Rho | 0.00591056 | -0.0210112 |

*Source: Own calculations using DerivaGem*

By the values of delta, it is noted that an increase by one unit in spot price determines an increase by 0.252167 RON in the call premium and a reduction by 0.7428446 RON in the put premium.

Gamma takes the value 0.2895241, which means that an increase in the share price by 1 RON (from 10 to 11) will increase the option value with 0.2895241 RON.

Vega is 0.01592383, which means that if the underlying volatility increase by one percentage point (from 22% to 23%), then both call and put premium will increase by 0.01592383.

Theta indicates the rate of change of the option premium with respect to the passage of time. The reduction of time to maturity by one day leads to a reduction by 0.002332 in call premium and by 0.0003694 in put premium.

Rho shows that if the interest rate increase by one percentage point, then call premium increase by 0.00591056, and put premium decrease by 0.0210112.

Considering the coefficients delta, gamma and theta defined above, equation can be rewritten with the Black-Scholes partial differential according to them. Thus, the relationship between delta, gamma and theta for a European option, in the Black-Scholes model assumptions is:

$$\Theta_c + rS\Delta_c + \frac{1}{2}\sigma^2 S^2 \Gamma_c = rc$$

$$\Theta_p + rS\Delta_p + \frac{1}{2}\sigma^2 S^2 \Gamma_p = rp$$

In addition, it is noted that these indicators interact and may not be regarded as separate entities.

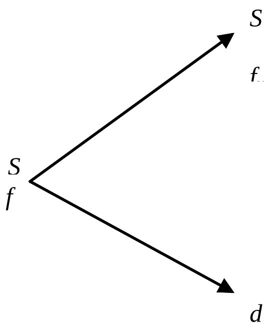
A higher volatility increases the delta for out of the money and at the money options and it brings down in the money options, property resulting from delta to measure the probability of exercising the option. Changing the gamma to the increased volatility is more pronounced for at the money options and lower for those out of the money.

## 3. A one-step binomial tree

Consider a call option on a non-dividend-paying stock. The assumptions of the model are the same for the Black-Scholes model, that the market id efficient, there are no transaction costs and no tax, securities are perfectly divisible, short selling is allowed, revenues generated by traded securities are remunerated at the risk free rate, *r*, which is constant, volatility remains constant throughout the life of the option. Add to this fact that the price of the underlying asset follows a binomial process in a time *T*, so this is the only hypothesis on the evolution of the underlying asset price. So, if at the time *0* the stock price is *S*, this can move up in *T* by *u* times with probability *p* or to move down with probability *1-p*. The process described is called the binomial multiplicative process.

The binomial model is based on building a risk free rate portfolio with a short position in a call option and a long position in Δ shares.

**A one-step binomial tree**




**Example no. 2***:* We propose to evaluate an European call option with a maturity of three months and the exercise price of 11 RON. A stock price is currently 10 RON. We suppose that at the end of three months the stock price will be either 12 RON or 9 RON. If the stock price turns out to be 12 RON, the value of the option will be 1 RON. If the stock price turns out to be 9 RON, the value of the option will be zero.

Consider a portfolio consisting on a short position in a call option and a long position in Δ shares. We calculate the value of Δ that makes the portfolio riskless. If the stock price moves up from 10 to 12 RON, the value of the option is 1RON, so that the total value of the portfolio is 12Δ-1. If the stock price moves down to 9 RON, the value of the portfolio will be 9Δ. The portfolio is riskless if the value of Δ is chosen so that the final value of the portfolio is the same for both alternatives. This means $12\Delta-1=9\Delta \rightarrow \Delta=0,333$.

The riskless portfolio contains 33 shares and one option. Whether the stock price moves up or down, the value of the portfolio is always 3 RON (12*0,333-1=9*0,33333≈3).

In the absence of arbitrage opportunities, riskles portfolios must earn the risk-free rate of interest. Suppose that the risk-free rate is 6% per annum. It follows that the present value of the portfolio is:

$$3\, e^{-0,06*3/12}=2,955$$

The value of the stock price today is 10 RON. If we denote the option price by *f*, then:

$$10x0,333-f=2,955$$

$$f=0,378$$

In conclusion, in the absence of arbitrage opportunities, the current value of the option must be *0,378 RON*. If the value of the option were more than *0,378 RON*, the portfolio would cost less than 2,955 RON and would earn more than the risk-free rate.

We can generalize the argument just presented by considering a stock whose current price is f. We denote with T the maturity of the option and we suppose that during the life of the option the stock price can either moves down to Sd or moves up to Su, where u>1 and d<1. The proportional increase in the stock price when is an up movement is u-1, and the proportional decrease when there is a down movement is 1-d. If the stock price moves up to *Su*,we suppose that the payoff from the option will be $f_u$ and if the stock price moves down to *Sd*, we suppose that the payoff from the option will be $f_d$.

We will calculate the value of Δ that makes the portfolio riskless. It follows that Δ can be chosen so that the final value of the portfolio be the same whether the price of the underlying asset increases or decreases during T.

$$Su\Delta-f_u = Sd\Delta-f_d \rightarrow \Delta=\frac{f_u - f_d}{S(u-d)}$$

In this case the portfolio is riskless and must earn the risk-free interest rate. The previous equation shows that Δ is the ratio of the change in the option price to the change in the stock price as we move between the nodes. If r is the risk-free rate, the present value of the portfolios:

$$(Su\Delta- f_u)e^{-rT}$$

The cost of setting up the portfolio is $S\Delta- f$.

It follows that:

$$S\Delta - f = (Su\Delta - f_u)e^{-rT}$$

$$f = S\Delta - (Su\Delta - f_u)e^{-rT}$$

Substituting for Δ and simplifying, we obtain:

$$f = e^{-rT}[p\, f_u + (1-p)\, f_d]$$

where $p = \dfrac{e^{rT} - d}{u - d}$

In the numerical example considered previously, u=1.2, d=0.9, r=0.06, T=0.25, $f_u=1$, $f_d=0$. It follows that:

$$p = \frac{e^{0.06x0.25} - 0.9}{1.2 - 0.9} = 0.38371$$

$$f = e^{-0.06x0.25}[0.38371x1 + (1-0.38371)x0] = 0.378$$

## 4. Generalized Binomial Model

The one-step binomial tree cand be exetended for a number of *n* periods, considering all possible states of the stock price, with *i* up movements and *n-i* down movements.

The stock price is initially S, the risk-free rate is r, and the length of the time stept is δ*t* years. Given the previous results we obtain:

$$f_u = e^{-r\delta t}[pf_{uu} + (1-p)f_{ud}]$$

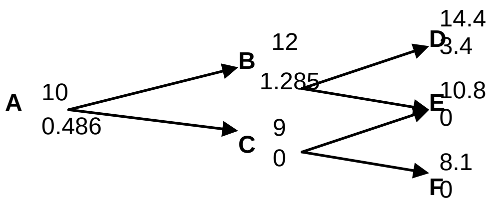
$$f_u = e^{-r\delta t}[pf_{ud} + (1-p)f_{dd}]$$

$$f = e^{-r\delta t}[pf_u + (1-p)f_d]$$

Substituting the first two equations into the last, we have:

$$f = e^{-2r\delta t}[p^2 f_{uu} + 2p(1-p)f_{ud} + (1-p)^2 f_{dd}]$$

**Example no. 3:** We consider the call option with the same characteristics as in the previous example.




Our objective is to calculate the option price at the initial node of the tree. We begin by setting the option price at the final nodes. At node D the stock price is 14.4, and the option price is 14.4-11=3.4. At nodes E and F, the option is out of the money an its value is zero. At node C, the option price is zero because node C leads to either node E or node F and at both nodes the option price is zero.

We will calculate the option price at node B. We know that u=1.2, d=0.9, r=0.06, T=0.25, so p=0.38371. It follows that the value of the option at node B is

$$e^{-0.06x0.25}[0.38371x3.4 + (1-0.38371)\, x0] = 1.285$$

It remains for us to calculate the option price at node A. Thus the value of the option is

$$e^{-0.06x0.25}[0.38371x1.285 + (1-0.38371)\, x0] = 0.486$$

The formula can be extended for n periods using the same mechanism with *i* up movements and *n-i* down movements of the stock price, for an European call option being:

$$c = e^{-n\, r\delta t} \sum_{i=0}^{n} \frac{n!}{i!(n-i)!} p^i (1-p)^{n-i} \max(u^i d^{\,n-i} S - X, 0)$$

In a similar manner, it can calculate for an European put option:

$$p = e^{-n\, r\delta t} \sum_{i=0}^{n} \frac{n!}{i!(n-i)!} p^i (1-p)^{n-i} \max(X - u^i d^{\,n-i} S, 0)$$

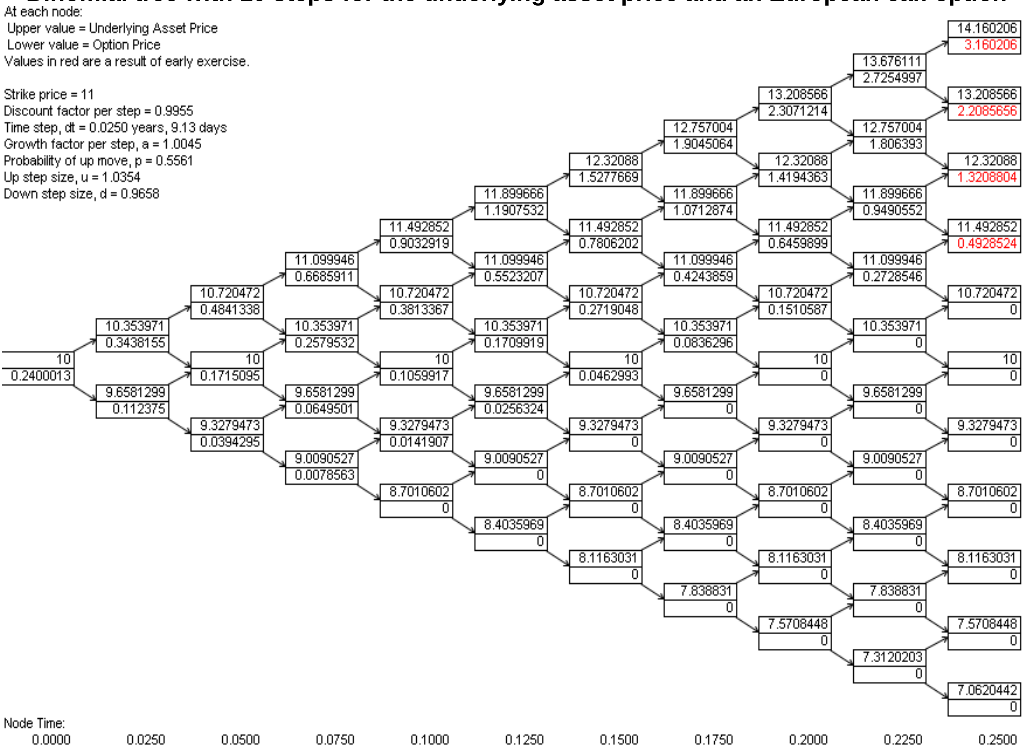
Previous formula describe some widely used algorithms especially in the valuation of the options which offer exercise opportunities before the maturity or whose underlying assets generate dividends.

Essentially, after choosing the number of periods for dividing the option life – usually 30 or more steps- it is built the binomial network for the underlying asset, following that the option price to be determined little by little starting with the final nodes of the network.

**Example no. 4:** We consider an European call option with a maturity of three months and the exercise price of 11 RON. A stock price is currently 10 RON, the risk free-rate is 8% per annum, the volatility is 22% per annum.

The figure below illustrates the binomial tree with 10 steps.

**Binomial tree with 10 steps for the underlying asset price and an European call option**




Determination of sensitivity coefficients with binomial model can be done by the transcription of definitions in dicrete form, considering the interval *δt*.

Thus, it obtains for a call option:

- **delta** $\Delta = \frac{\delta f}{\delta S}$

For the first step, the coefficient can be expressed as:

- **gamma** $\Gamma = \frac{\delta^2 f}{\delta S^2}$

As we can see gamma can be calculated with a delay of two periods *δt* and for the first two periods its value can be approximated:

$$\Gamma = \frac{\frac{f_{22} - f_{21}}{Su^2 - S} - \frac{f_{21} - f_{20}}{S - Sd^2}}{\frac{Su^2 - Sd^2}{2}}$$

- **theta** $\Theta = \frac{\delta f}{\delta t}$

In the event that this period the underlying asset price, or the other parametters remain constant, the coefficient can be approximated for the first two nodes:

$$\Theta = \frac{f_{21} - f_{20}}{2\delta t}$$

- **vega** $K = \frac{\delta f}{\delta \sigma} = \frac{f^* - f}{\delta \sigma}$

where *f** is the value of the option for the volatility *σ+δσ.*

- **rho** $P = \frac{\delta f}{\delta r} = \frac{f^* - f}{\delta \sigma}$

Estimation of sensitivity coefficients in the manner described above is useful for following up hedging transactions, whose continuoully sequence is not possible.

**Example no. 5:** We consider an European call option with a maturity of three months and the exercise price of 11 RON. A stock price is currently 10 RON, the risk free-rate is 8% per annum, the volatility is 22% per annum.

Sensitivity coefficients have the values below in Black-Scholes version and in binomial version with 2, 10, 20, 30, 50, 100 steps.

**Calculation of sensitivity coefficients. Black-Scholes Model vs. Binomial Model**

| | Black-Sholes Model | Binomial Model n=2 | Binomial Model n=10 | Binomial Model n=20 | Binomial Model n=30 | Binomial Model n=50 | Binomial Model n=100 |
|---|---|---|---|---|---|---|---|
| Delta | 0.264465 | 0.236763 | 0.260251 | 0.257564 | 0.262967 | 0.261829 | 0.263946 |
| Gamma | 0.297461 | 0.25986 | 0.302752 | 0.301808 | 0.298978 | 0.299448 | 0.29793 |
| Vega | 0.016360 | 0.021132 | 0.015410 | 0.019129 | 0.015711 | 0.016667 | 0.015817 |
| Theta | -0.00251 | -0.00218 | -0.00253 | -0.00252 | -0.00252 | -0.00252 | -0.00251 |
| Rho | 0.006193 | 0.005455 | 0.006156 | 0.006113 | 0.006235 | 0.006216 | 0.006264 |

*Source: Own calculations using DerivaGem*

It can be seen that the the differences are becoming smaller as the number of steps increases.

Similarly, sensitivity coefficients can be calcaulated for a put option.

## 5. Conclusions

Used both for hedging risks and for speculation, the options are found in the portfolios of various institutions - from hedge funds and financial institutions, corporations or individual investors.

Options have demonstrated successfully their important role in the financial markets. In an ideal setting, derivatives pricing theory provides a framework in which the risks inherent to an option’s position can be minimized or eliminated via a dynamic hedging strategy. In practice, however, the effectiveness of such strategy can be limited due to the lack of available hedging instruments and market microstructure issues such as transaction costs and market illiquidity. In addition to the generally high leverage or sensitivity of derivatives value to the change of underlying asset value, a unique and yet very important risk to options is the so-called *model risk* that arises whenever derivatives pricing and/or hedging strategies are based on a miss-specified model.

Options prices, as those assets that constitute their support, are affected by several factors. Knowing these factors and how they affect the value of options is essential to use as a tool for financial risk management. Sensitivity coefficients of the options premium measure the response of their price to each of the factors influencing it, providing an image of the risk of a position on an option.

The previous examples have shown that the difference in the calculation by the two models of options price and their sensitivity coefficients disappear as the number of binomial tree steps increases.

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